Finance Basics

Time Value of Money & the Yield Curve

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Agenda

I. Finance Basics

- Time Value of Money
- Yield Curve

II. Bond Math Primer

- Terminology
- Types of Bonds
- Bond Pricing

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Finance Basics

Simple and Compound Interest

Interest Earnings 7.00%

Initial Investment 1,000.00

Compounding interest results in higher ending balances – earning interest on previously earned interest

Simple Interest						
Period	Beginning Balance	Initial investment		Interest	Ending Balance
1	1,000	1,000	x .07 =	70	1,070
2	1,070	1,000	x .07 =	70	1,140
3	1,140	1,000	x .07 =	70	1,210
4	1,210	1,000	x .07 =	70	1,280
5	1,280	1,000	x .07 =	70	1,350
6	1,350	1,000	x .07 =	70	1,420
7	1,420	1,000	x .07 =	70	1,490
8	1,490	1,000	x .07 =	70	1,560
9	1,560	1,000	x .07 =	70	1,630
10	1,630	1,000	x .07 =	70	1,700
Total				**700**	

Compound Interest				
Period	Beginning Balance		Interest	Ending Balance
1	1,000	x .07 =	70	1,070
2	1,070	x .07 =	75	1,145
3	1,145	x .07 =	80	1,225
4	1,225	x .07 =	86	1,311
5	1,311	x .07 =	92	1,403
6	1,403	x .07 =	98	1,501
7	1,501	x .07 =	105	1,606
8	1,606	x .07 =	112	1,718
9	1,718	x .07 =	120	1,838
10	1,838	x .07 =	129	1,967
Total			**967**	

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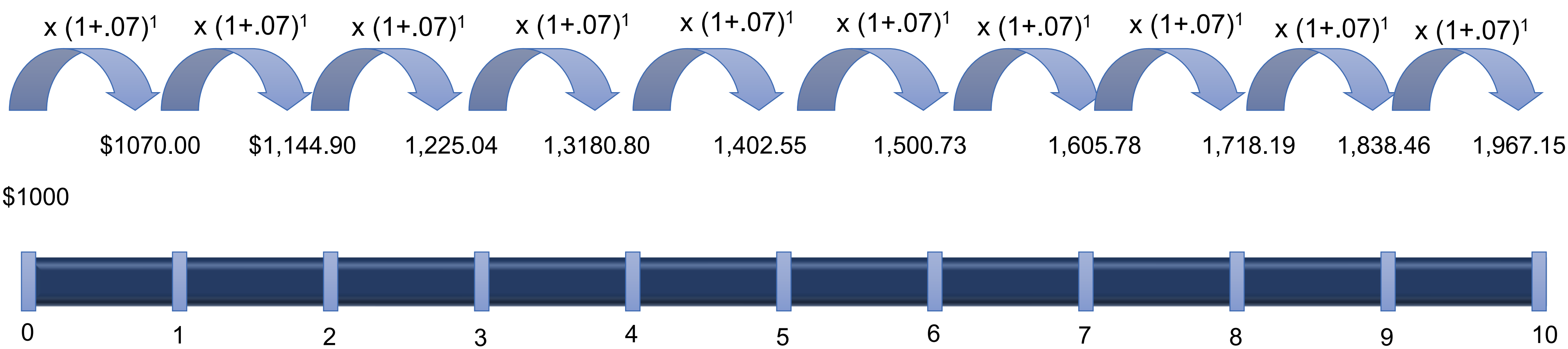
Time Value of Money – Future Value

- $1,000 in the future is worth *less* when stated as a current value.
  - Said differently, $1,000 in hand today is worth more than the same $1,000 in the future.

$$FV_n = PV_0(1+r)^n$$

Time Value of Money – Future Value (cont'd)

Future Value of $1,000 ***compounded*** for 1-10 periods at 7%



$$=1000(1+.07)^1+(1+.07)^1+(1+.07)^1+(1+.07)^1+(1+.07)^1+(1+.07)^1+(1+.07)^1+(1+.07)^1+(1+.07)^1+(1+.07)^1$$

$$=1000(1+.07)^{10}$$

$$=PV_0\ (1+r)^n$$

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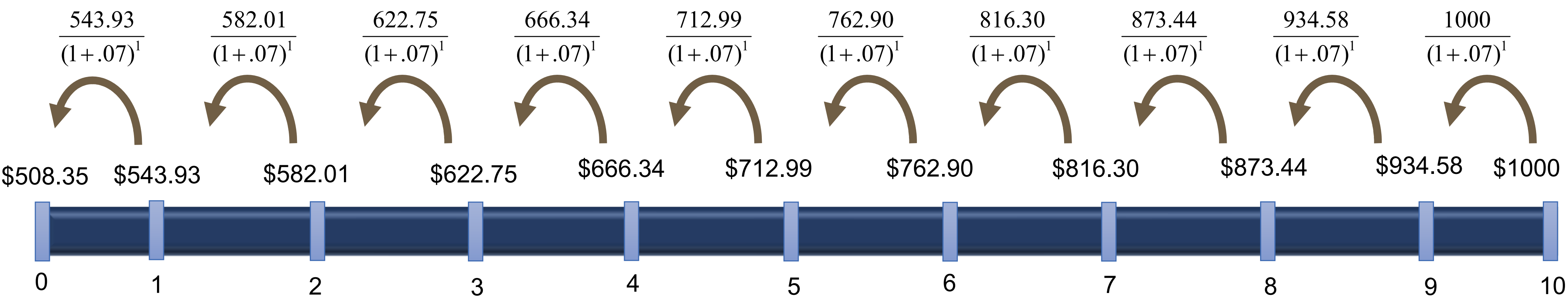
Time Value of Money – Present Value

- $1,000 today will be worth *more* when stated as a value in the future.

$$PV_0 = \frac{FV_n}{(1+r)^n}$$

Time Value of Money – Present Value (cont'd)

Present Value of $1,000 compounded for 1-10 periods at 7%



$$= \frac{1000}{(1.07)^1(1.07)^1(1.07)^1(1.07)^1(1.07)^1(1.07)^1(1.07)^1(1.07)^1(1.07)^1(1.07)^1}$$

$$= \frac{FV_n}{(1+r)^1(1+r)^1(1+r)^1(1+r)^1(1+r)^1(1+r)^1(1+r)^1(1+r)^1(1+r)^1(1+r)^1}$$

$$= \frac{FV_n}{(1+r)^n}$$

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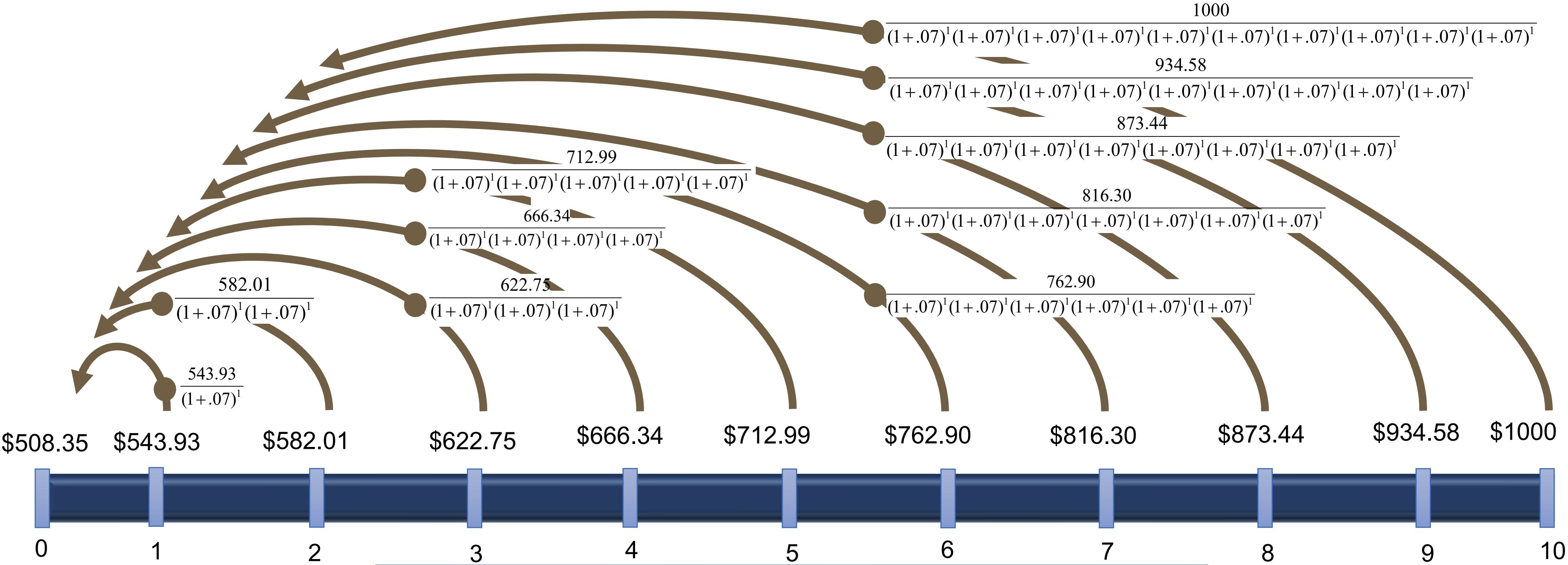
Time Value of Money – Present Value (cont'd)

Present Value of $1,000 compounded for 1-10 periods at 7%



$$= \frac{1000}{(1.07)^1(1.07)^1(1.07)^1(1.07)^1(1.07)^1(1.07)^1(1.07)^1(1.07)^1(1.07)^1(1.07)^1}$$

$$= \frac{FV}{(1+r)^1(1+r)^1(1+r)^1(1+r)^1(1+r)^1(1+r)^1(1+r)^1(1+r)^1(1+r)^1(1+r)^1}$$

$$= \frac{FV}{(1+r)^n}$$

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Future Value and Present Value

- Assuming the cost of gum is $1.15 today and an inflation rate of 1.5%, what will the cost of gum be in 15 years?

- Assuming inflation at 1.5% what was the cost of gum 15 years ago?

15 years ← Today → 15 years

$$\frac{\$1.15}{(1+.015)^{15}} = \$0.92$$

Today

$$\$1.15 \text{ x } (1+.015)^{15} = \$1.44$$

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Poll question 1

Is the following statement True or False: $100 dollars today holds the same value to the holder as $100 dollars 10 years from today.

A. True

B. False

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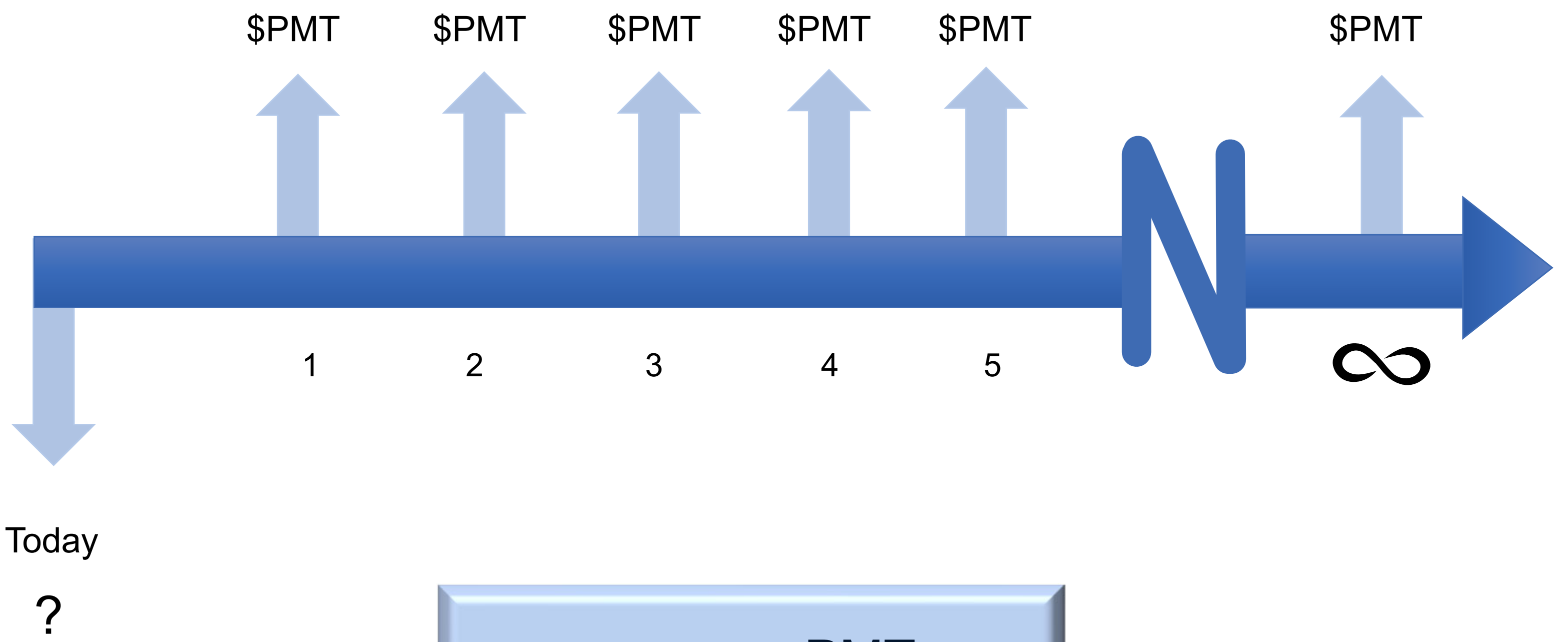
Perpetuity

- Perpetuity – Equal payments at equal intervals forever



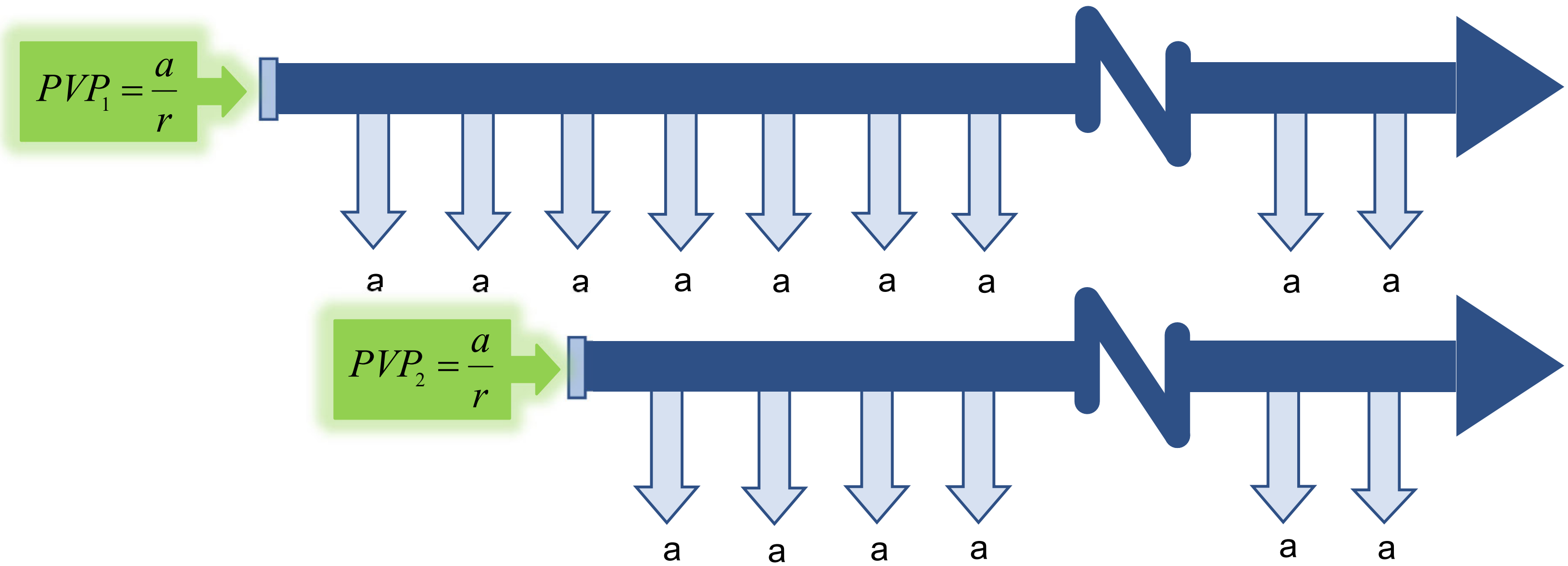
$$PVP = \frac{PMT}{r}$$

Time Value of Money — Annuity

An annuity is the difference between two perpetuities.

In General,

$$PVA = \frac{PMT}{r}\left[1 - \frac{1}{(1+r)^n}\right]$$



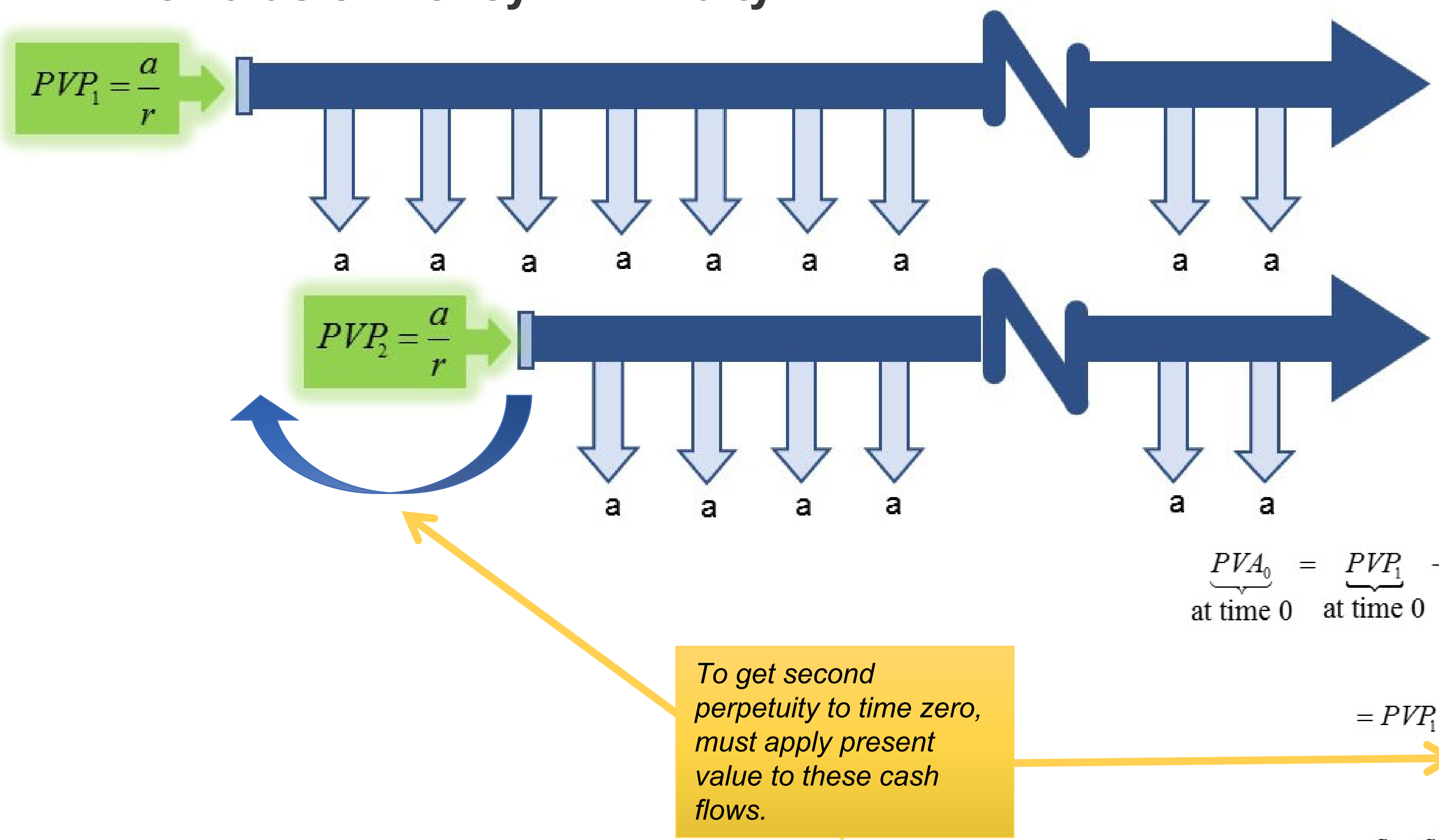
So,

PV Annuity $= PVP_1 - PVP_2$

$= \frac{a}{r} - \frac{a}{r}$

$= 0$ **WRONG! ☹**

Time Value of Money — Annuity



$$\underbrace{PVA_0}_{\text{at time 0}} = \underbrace{PVP_1}_{\text{at time 0}} - \underbrace{PVP_2}_{\text{at time 0}}$$

$$= PVP_1 - \frac{PVP_2}{(1+r)^n}$$

$$= \frac{a}{r} - \frac{a}{r}\left(\frac{1}{(1+r)^n}\right)$$

$$PVA = \frac{a}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$

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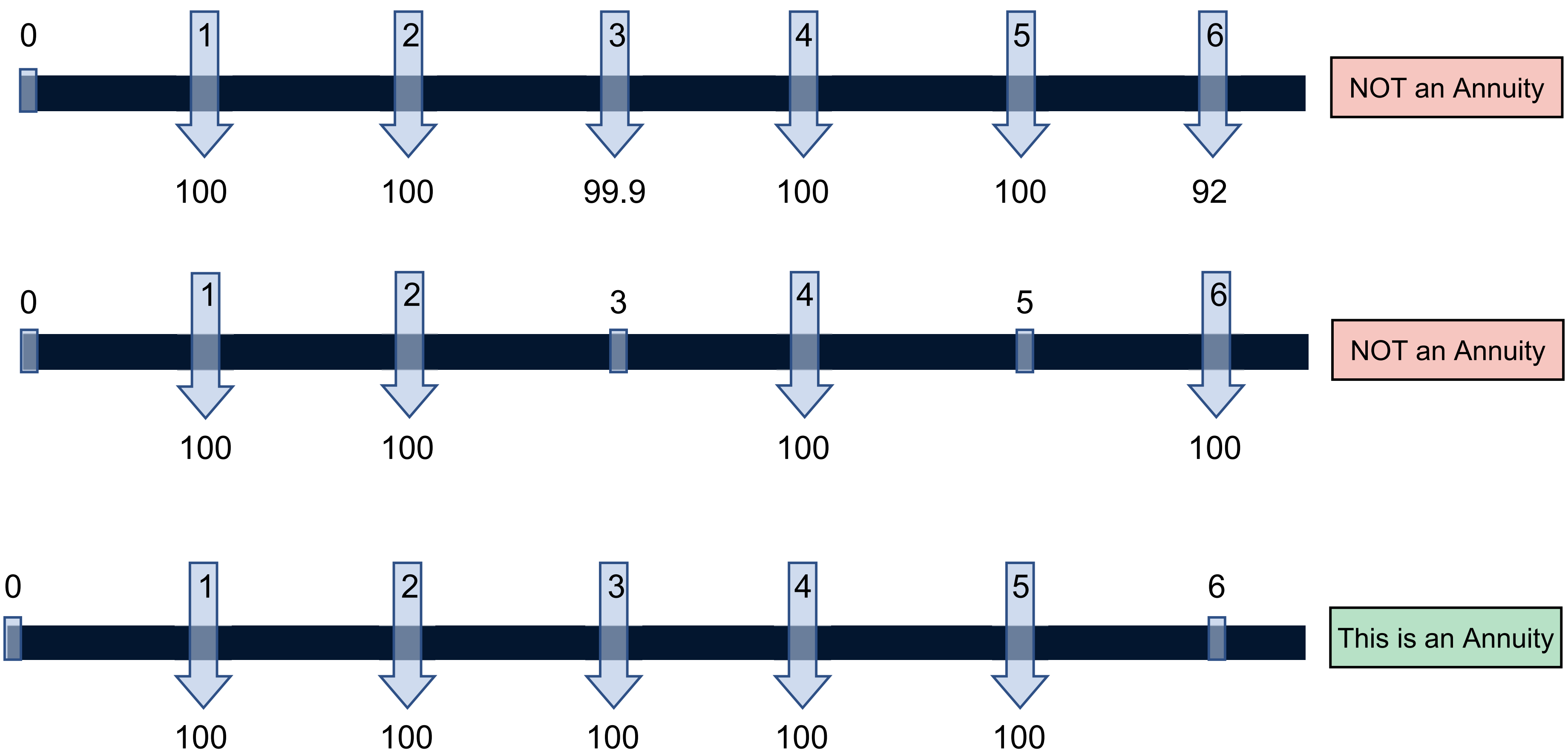
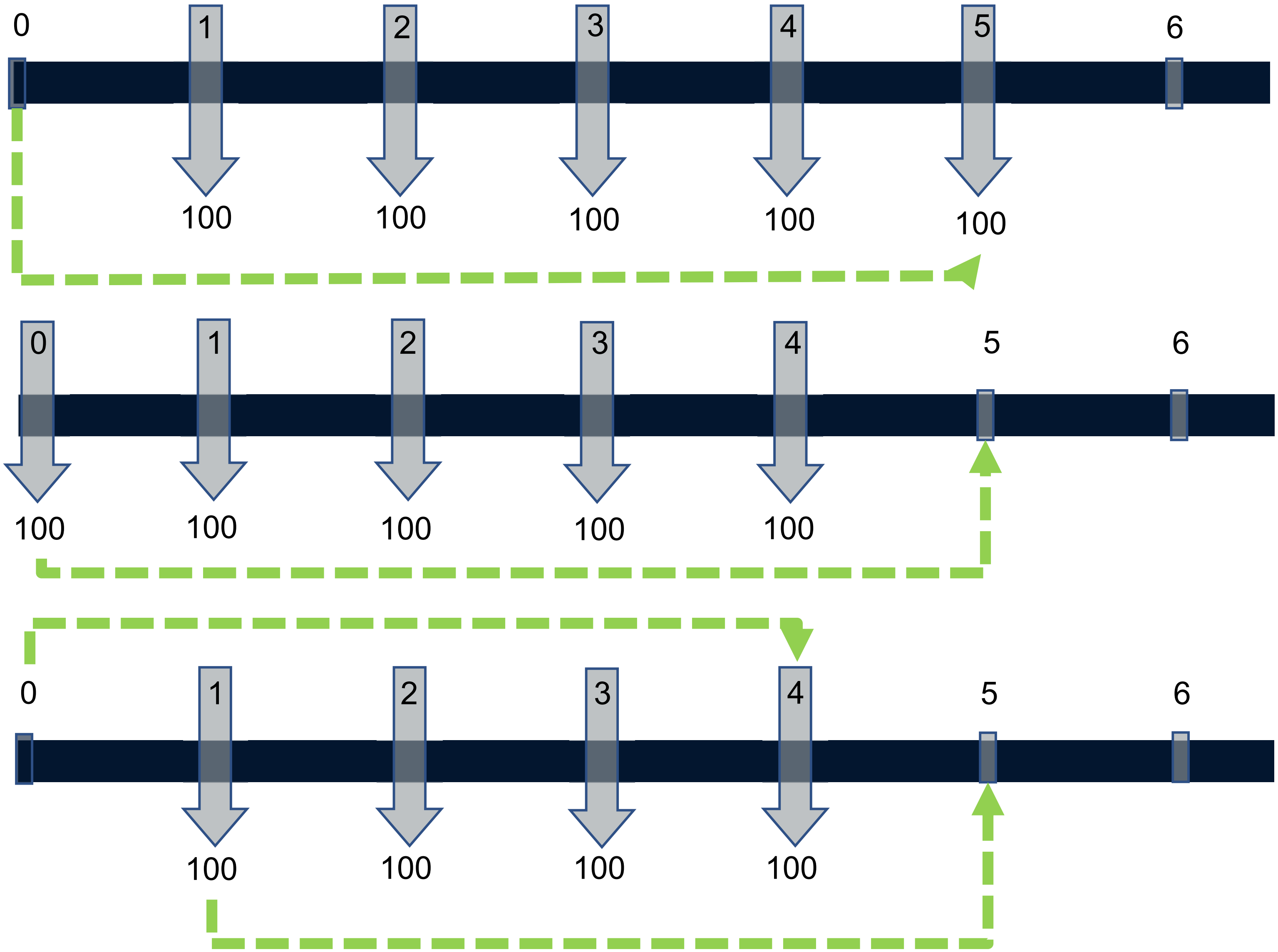
Time Value of Money — Annuity

A payment stream with equal payments over equal periods of time is called an ***ANNUITY***.



Time Value of Money — Annuity



Annuity in ARREARS
(Relative to time 0)
Note: n = 5

Annuity in ADVANCE
(Relative to time 0)
Note: n = 5

Either an Annuity in ARREARS (n = 4, starting at 0) or ADVANCE (n = 4, starting at 1)

Poll question 2

Which of the below represents an annuity?

A. $100 payments every month of the year except for July

B. $100 payments every month from February through and including October

C. $100 payments every month from February through and including June, and $120 payments every month from July through and including October

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Term Structure of Spot Rates

Until now, we have discounted all the cash flows at a single rate (r). r = 7.00%

Period	Cash Flow	Applicable Rate	Discount Factor $PV_n = 1/(1+r)^n$	Present Value
1	1,000	7.00%	0.93458	934.58
2	1,000	7.00%	0.87344	873.44
3	1,000	7.00%	0.81630	816.30
4	1,000	7.00%	0.76290	762.90
5	1,000	7.00%	0.71299	712.99
6	1,000	7.00%	0.66634	666.34
7	1,000	7.00%	0.62275	622.75
8	1,000	7.00%	0.58201	582.01
9	1,000	7.00%	0.54393	543.93
10	1,000	7.00%	0.50835	508.35
			Total	**7,023.58**

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Term Structure of Interest Rates

In actuality, each cash flow should probably be discounted at the rate applicable to that period.

Period	Cash Flow	Applicable Rate	Discount Factor $PV_n = 1/(1+r)^n$	Present Value
1	1,000	3.000%	0.97087	970.87
2	1,000	4.000%	0.92456	924.56
3	1,000	5.000%	0.86384	863.84
4	1,000	6.000%	0.79209	792.09
5	1,000	7.000%	0.71299	712.99
6	1,000	8.000%	0.63017	630.17
7	1,000	9.000%	0.54703	547.03
8	1,000	10.000%	0.46651	466.51
9	1,000	11.000%	0.39092	390.92
10	1,000	12.000%	0.32197	321.97
			Total	**6,620.96**

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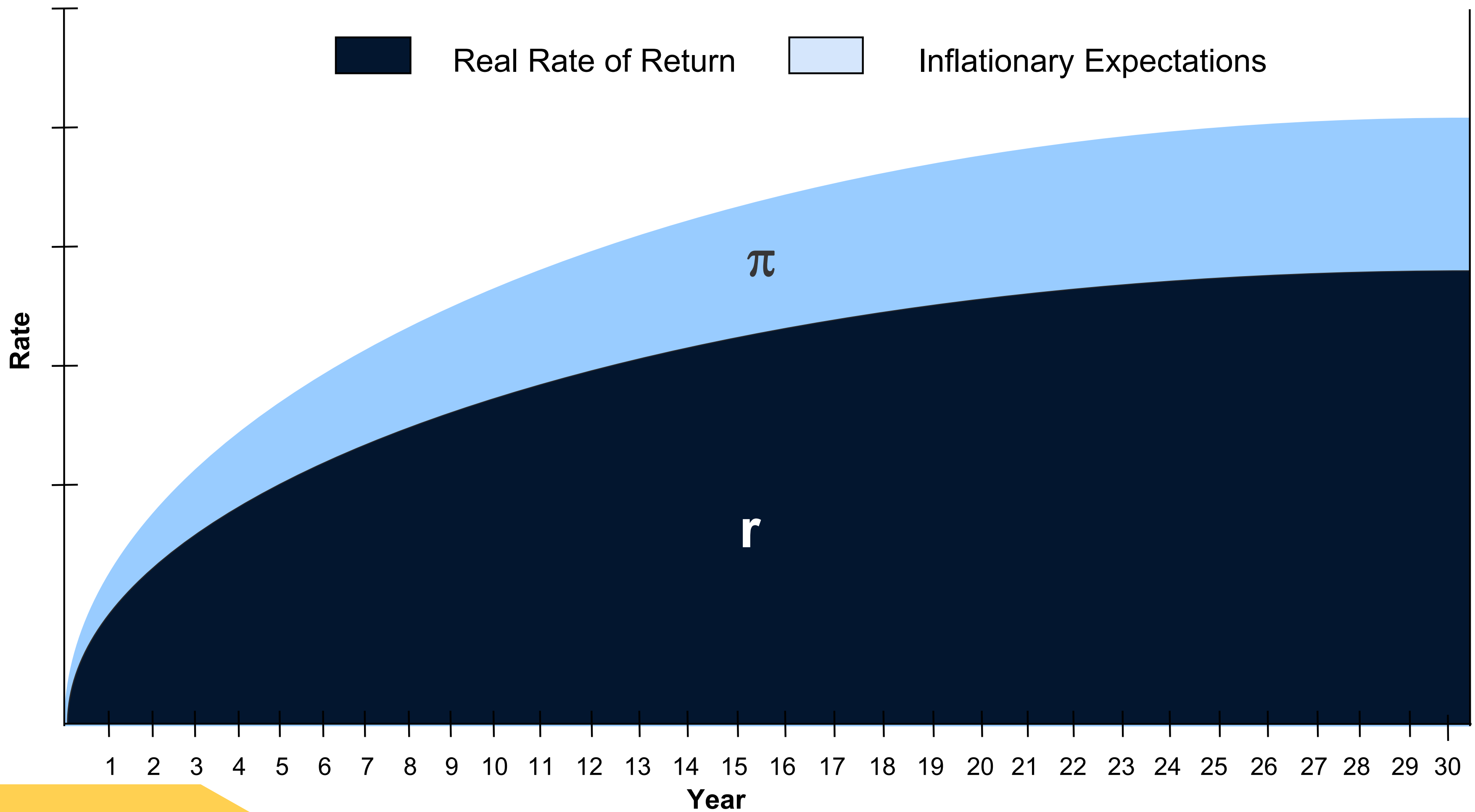
Yield Curve

Fisher Equation

$$R = r + \pi$$



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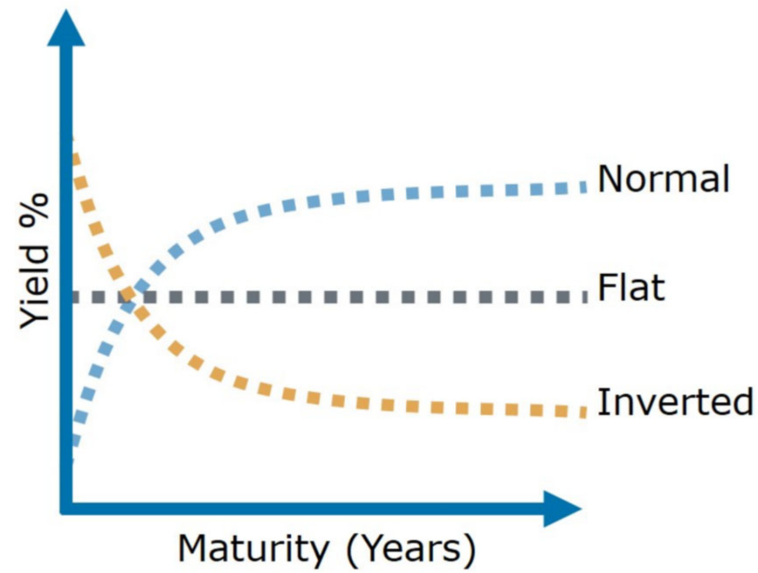
Yield Curve (cont'd)

- A **normal yield curve** forms during market conditions where investors generally believe that there will be no significant changes to the economy (i.e., no inflation).
- Investors expect long term instruments to offer higher yields than short term instruments – this includes the assumption of additional risk for longer investments and the expectation for greater yield for taking on that risk.



- A **flat yield curve** structure reveals that there may be some signs that short-term rates will rise and others that long-term rates will fall.
- In an **inverted yield curve** market, which is rare, investors assume long term rates will decline
- This curve also implies that investors expect to receive less compensation for assuming more risk
- Some investors interpret an inverted yield curve as a signal that the economy is going to experience a slow down or is experiencing a slow down.

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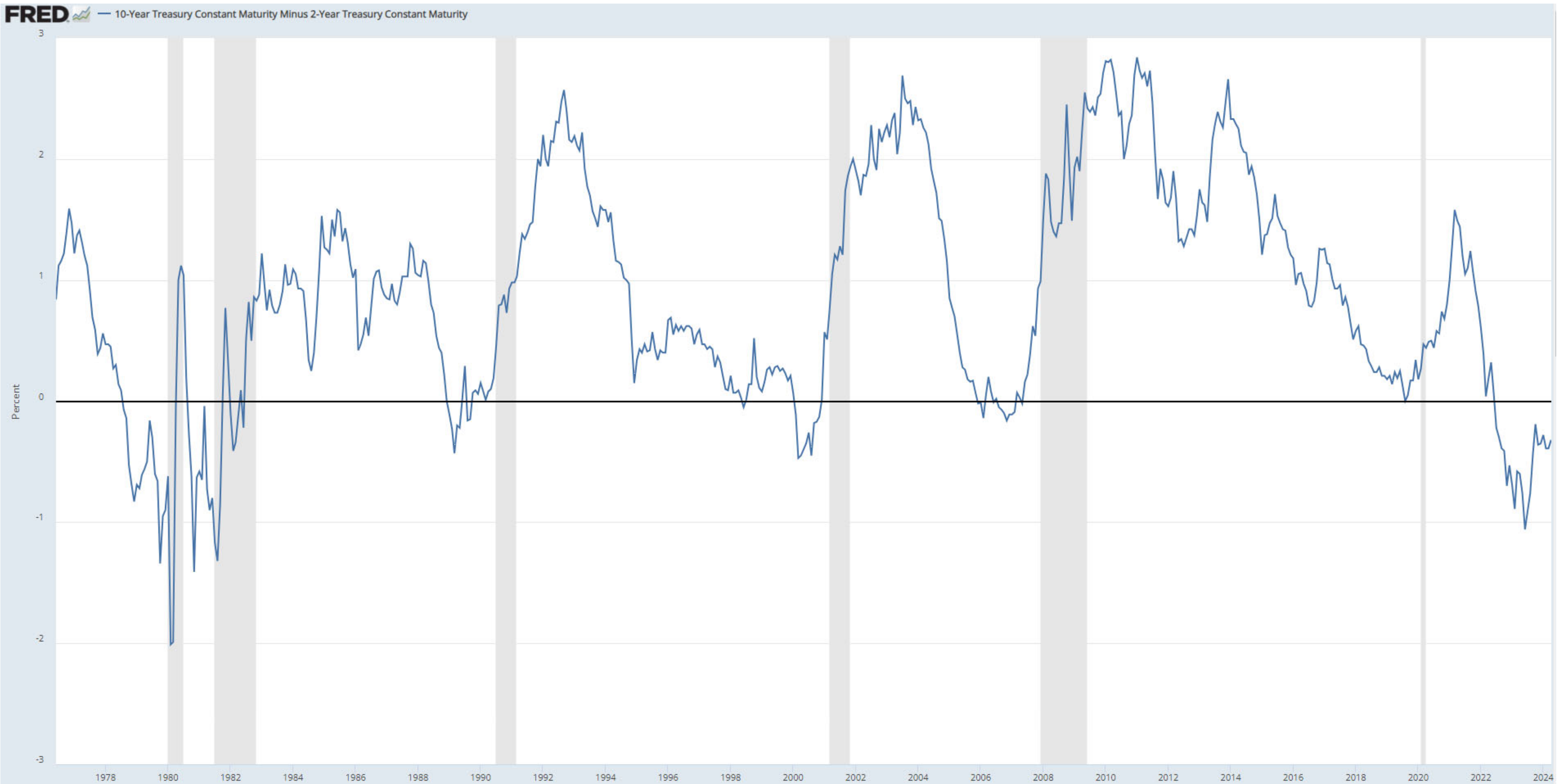
Inverted Yield Curves and US Recessions



Source: FRED, Federal Reserve Bank of St. Louis

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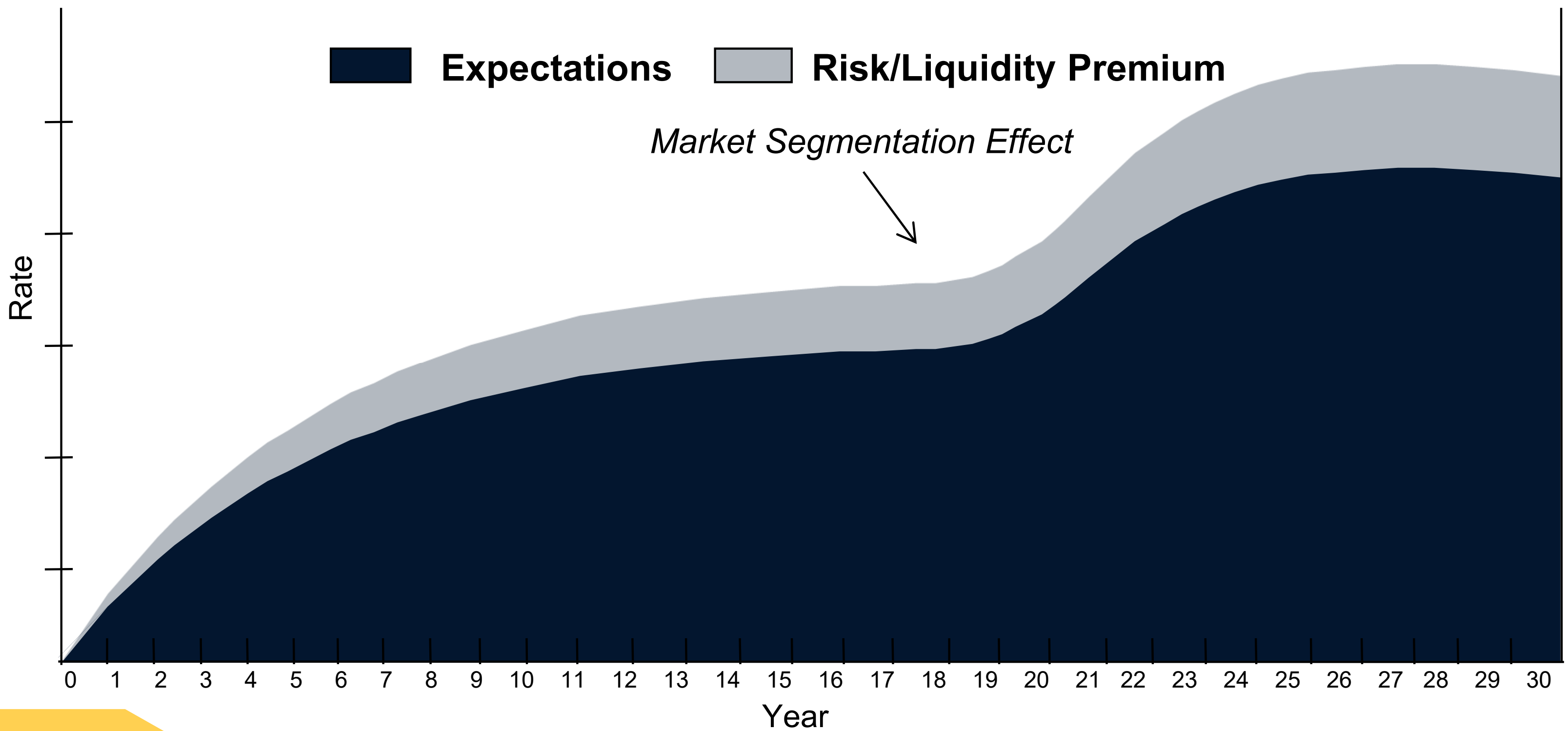
Yield Curve (cont'd)

The yield curve may be influenced by exceptionally strong or weak investor demand for securities of a given maturity. For example, banks may prefer to purchase securities with terms less than 20 years, eschewing what may be relatively higher interest rates at longer terms. The effect would be to increase demand in the 1 - 20 year range, meaning investors would accept/borrowers would receive lower interest rates on securities maturing during this period. Conversely, a lack of demand after 20 years could elevate interest rates in this segment of the curve.



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Poll question 3

Historically, inverted US Treasury yield curves often are followed by:

A. Sustained economic growth

B. Population growth

C. Economic recession

D. Strong corporate earnings reports

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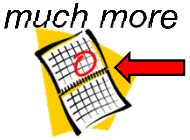
Bond Math Primer

Why Issue Bonds?

- States, cities, counties and other public authorities are responsible for funding public projects such as the construction and upkeep of schools, hospitals, highways, sewers, and universities
- How should issuers fund these capital projects?
  - **Option 1: Use that treasure chest of funds that's been sitting around.**
    - *What treasure chest of funds???*

  - **Option 2: Save up money (maybe from a newly instituted tax) for a long period of time (20 to 30 years) and then build the project once the necessary amount has been saved (i.e., "pay as you go" funding).**
    - *Problem: Issuer needs the project now, the project may also be much more expensive in 20 to 30 years.*



    - *Problem: Unfair - Those that are taxed to fund the project should also be those that benefit from/ use the project (i.e., generational transfer).*

  - **Option 3: Issue Bonds.**
    - Issuers can procure funds today to build the project they need by borrowing money through a bond issuance.
    - The debt service (i.e., principal and interest) on the bonds is paid by the users of the project (i.e., tax-payers, toll-payers, rate-payers).

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Tax Exemption

- The IRS deems bonds that are issued for qualified public projects by municipal governments (e.g., state and local governments) and non-profits entities (e.g., school districts, higher education, toll and transit, airports, public power, health care, etc.) tax-exempt.
  - The interest income on these bonds are exempt from federal income taxes.
  - Many are also exempt from state income taxes, for owners that reside within those states.
- Why? Because the capital projects funded by these bonds are for the good of the public.
- Due to these exemptions, tax-exempt bonds typically carry lower interest rates than comparable, taxable bonds.

	Taxable Bond	Tax-exempt Bond
Market Interest Rate	10.00%	6.70%
Less Taxes	(3.30%)	0.00%
Effective Interest Rate	6.70%	6.70%

Investors are willing to receive a lower rate since they are not required to pay taxes on interest income. Their net effective rate equalizes.

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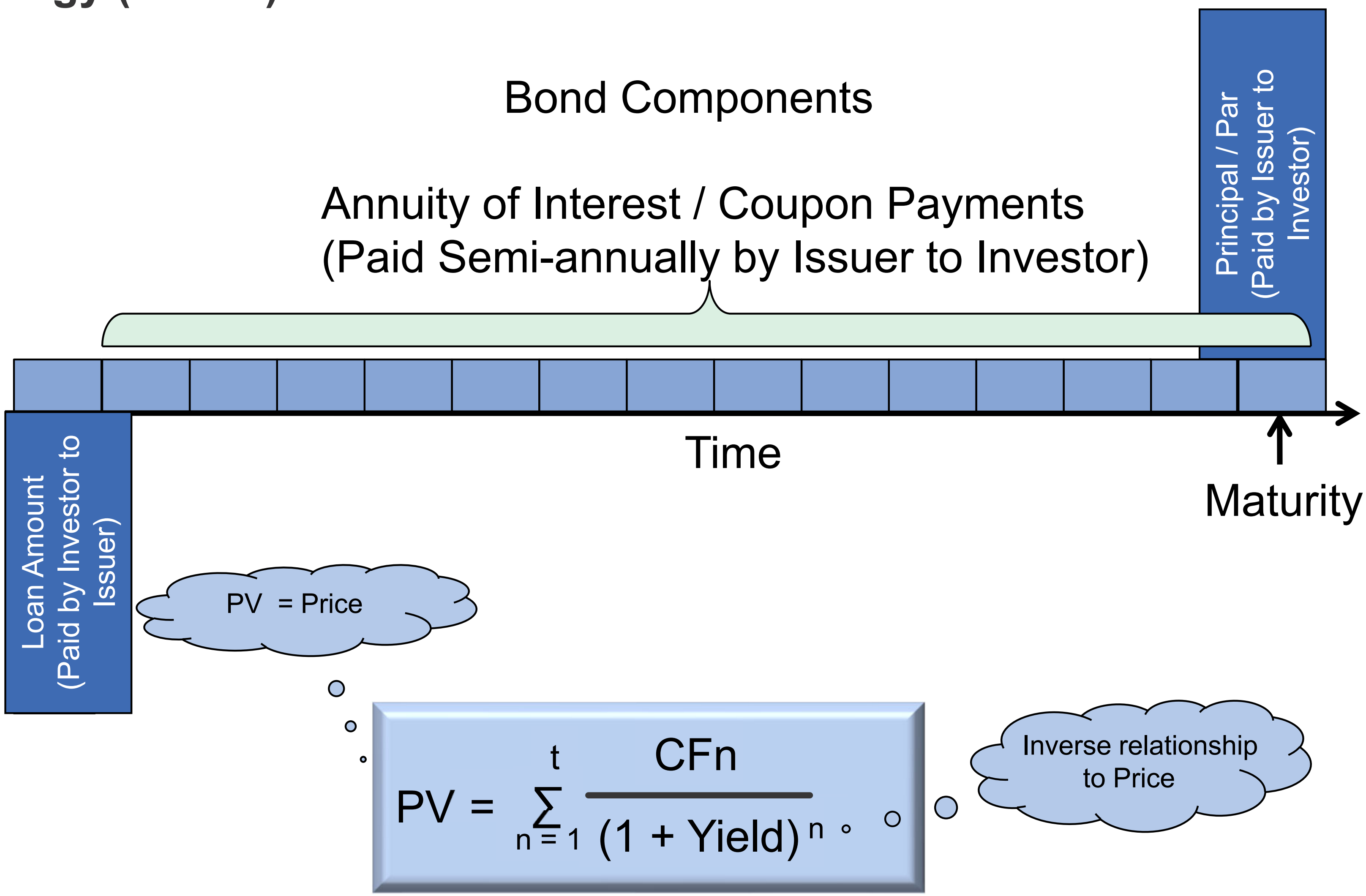
Terminology

- A Bond is evidence of a loan
  - Buyer of the Bond is the lender or investor
  - Seller of the Bond is the borrower or the issuer or the obligor
- Principal or Face Amount or Par Amount
  - Amount of loan
- Maturity date
  - Repayment date of loan – may be spread across multiple years
- Nominal or Coupon rate
  - Interest rate paid periodically on the loan
  - Usually expressed as a percentage of par amount
- Price
  - Amount a lender will lend in consideration of future receipt of principal and interest payments
- Yield
  - Single rate that sets the PV of the future principal and interest payments equal to the initial price

Terminology (cont'd)



Poll question 4

Tax-exempt status on municipal bonds is provided by the IRS in order to:

A. Lower the net cost of borrowing for issuers that finance public projects.

B. Create a greater demand for municipal bonds amongst investors.

C. Provide an incentive for governments to build and maintain public facilities.

D. All of the above.

E. None of the above.

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Pricing a Bond

$$\text{Price of a Bond} = PV_{\text{Cashflows}}$$

$$PV_{\text{Cashflows}} = PV_{\text{Interest Annuity}} + PV_{\text{Principal}}$$

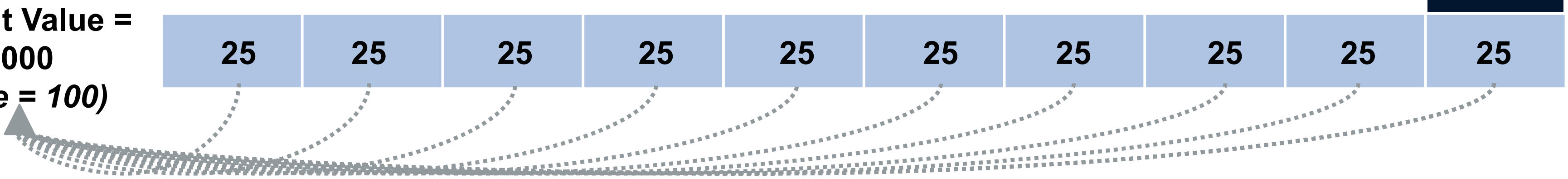
therefore

$$\text{Price of a Bond} = PV_{\text{Interest Annuity}} + PV_{\text{Principal}}$$

Par Bond

Par	1,000
Coupon	5.00% (2.5% paid semi-annually)
Yield	5.00%
Years	5

Present Value = 1,000
(Price = 100)



COUPON = YIELD

Bond Cash Flows

- Let's assume a municipality needed to borrow $10 million for a capital project (Issuance Date: 7/1/2023)
- To make it simple, let's just assume that there were 10 investors and each investor lent the municipality $1 million over various periods

Investor #	Lending Term	Repayment Date	Loan Amount	Coupon	Semi-annual Interest Payment
1	1 yr	7/1/2024	$1,000,000	3.000%	$15,000
2	2 yrs	7/1/2025	$1,000,000	3.125%	$15,625
3	3 yrs	7/1/2026	$1,000,000	3.250%	$16,250
4	4 yrs	7/1/2027	$1,000,000	3.375%	$16,875
5	5 yrs	7/1/2028	$1,000,000	3.500%	$17,500
6	6 yrs	7/1/2029	$1,000,000	3.625%	$18,125
7	7 yrs	7/1/2030	$1,000,000	3.750%	$18,750
8	8 yrs	7/1/2031	$1,000,000	3.875%	$19,375
9	9 yrs	7/1/2032	$1,000,000	4.000%	$20,000
10	10 yrs	7/1/2033	$1,000,000	4.125%	$20,625

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Par Cash Flows

Date	Principal	Coupon	Interest	Debt Service
1/1/2024	0	--	178,125	178,125
7/1/2024	1,000,000	3.000%	178,125	1,178,125
1/1/2025	0	--	163,125	163,125
7/1/2025	1,000,000	3.125%	163,125	1,163,125
1/1/2026	0	--	147,500	147,500
7/1/2026	1,000,000	3.250%	147,500	1,147,500
1/1/2027	0	--	131,250	131,250
7/1/2027	1,000,000	3.375%	131,250	1,131,250
1/1/2028	0	--	114,375	114,375
7/1/2028	1,000,000	3.500%	114,375	1,114,375
1/1/2029	0	--	96,875	96,875
7/1/2029	1,000,000	3.625%	96,875	1,096,875
1/1/2030	0	--	78,750	78,750
7/1/2030	1,000,000	3.750%	78,750	1,078,750
1/1/2031	0	--	60,000	60,000
7/1/2031	1,000,000	3.875%	60,000	1,060,000
1/1/2032	0	--	40,625	40,625
7/1/2032	1,000,000	4.000%	40,625	1,040,625
1/1/2033	0	--	20,625	20,625
7/1/2033	1,000,000	4.125%	20,625	1,020,625

The yield is determined by finding the rate that sets the PV of Debt Service and the upfront proceeds equal to one another.

0.00 <==PV of Cashflows minus Upfront Proceeds

3.00% <==yield

Date	Principal	Coupon	Interest	Debt Service	PV of Debt Service
1/1/2024	0	--	15,000	15,000	14,778
7/1/2024	1,000,000	3.000%	15,000	1,015,000	985,222
	Issuance Date:	**7/1/2023**		**YIELD:**	3.0000%
PROCEEDS	**1,000,000**		**30,000**	**1,030,000**	**1,000,000**

Pricing a Bond (cont'd)

$$\text{Price of a Bond} = PV_{\text{Cashflows}}$$

$$PV_{\text{Cashflows}} = PV_{\text{ Interest Annuity}} + PV_{\text{Principal}}$$

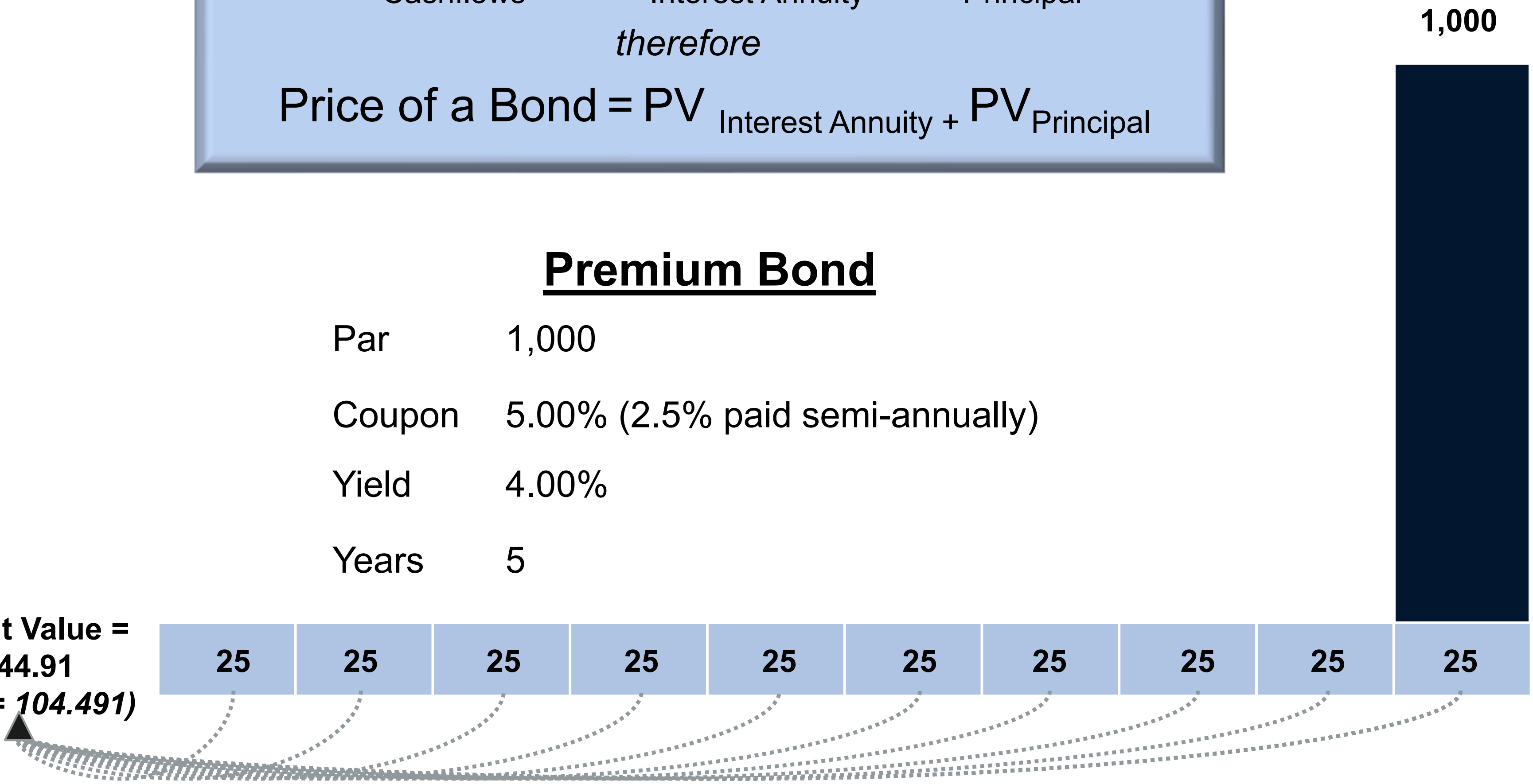
therefore

$$\text{Price of a Bond} = PV_{\text{ Interest Annuity}} + PV_{\text{Principal}}$$

Premium Bond

Par	1,000
Coupon	5.00% (2.5% paid semi-annually)
Yield	4.00%
Years	5

Present Value = 1,044.91
(Price = 104.491)



COUPON > YIELD

Premium Cash Flows

Date	Principal	Coupon	Interest	Debt Service
1/1/2024	0	--	178,125	178,125
7/1/2024	1,000,000	3.000%	178,125	1,178,125
1/1/2025	0	--	163,125	163,125
7/1/2025	1,000,000	3.125%	163,125	1,163,125
1/1/2026	0	--	147,500	147,500
7/1/2026	1,000,000	3.250%	147,500	1,147,500
1/1/2027	0	--	131,250	131,250
7/1/2027	1,000,000	3.375%	131,250	1,131,250
1/1/2028	0	--	114,375	114,375
7/1/2028	1,000,000	3.500%	114,375	1,114,375
1/1/2029	0	--	96,875	96,875
7/1/2029	1,000,000	3.625%	96,875	1,096,875
1/1/2030	0	--	78,750	78,750
7/1/2030	1,000,000	3.750%	78,750	1,078,750
1/1/2031	0	--	60,000	60,000
7/1/2031	1,000,000	3.875%	60,000	1,060,000
1/1/2032	0	--	40,625	40,625
7/1/2032	1,000,000	4.000%	40,625	1,040,625
1/1/2033	0	--	20,625	20,625
7/1/2033	1,000,000	4.125%	20,625	1,020,625

The yield is determined by finding the rate that sets the PV of Debt Service and the upfront proceeds equal to one another.

0.00 <==PV of Cashflows minus Upfront Proceeds

1.985% <==yield

Date	Principal	Coupon	Interest	Debt Service	PV of Debt Service
1/1/2024	0	--	15,000	15,000	14,853
7/1/2024	1,000,000	3.000%	15,000	1,015,000	995,147
	Issuance Date:	**7/1/2023**		**YIELD:**	1.9851%
PROCEEDS	**1,010,000**		**30,000**	**1,030,000**	**1,010,000**

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Pricing a Bond (cont'd)

$$\text{Price of a Bond} = PV_{Cashflows}$$

$$PV_{Cashflows} = PV_{\text{Interest Annuity}} + PV_{Principal}$$

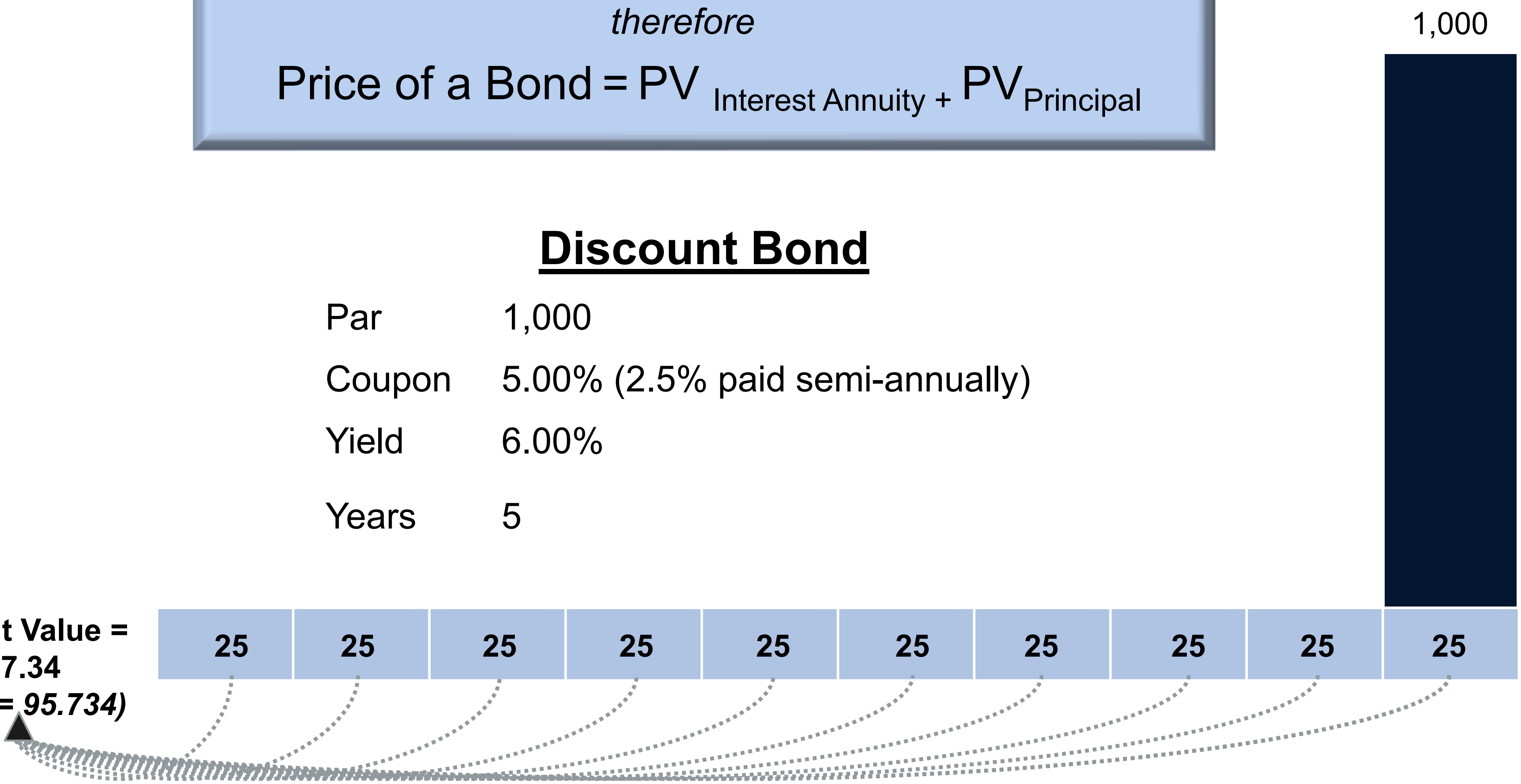
therefore

$$\text{Price of a Bond} = PV_{\text{Interest Annuity}} + PV_{Principal}$$

Discount Bond

Par	1,000
Coupon	5.00% (2.5% paid semi-annually)
Yield	6.00%
Years	5

Present Value = 957.34
(Price = 95.734)



COUPON < YIELD

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Discount Cash Flows

Date	Principal	Coupon	Interest	Debt Service
1/1/2024	0	--	178,125	178,125
7/1/2024	1,000,000	3.000%	178,125	1,178,125
1/1/2025	0	--	163,125	163,125
7/1/2025	1,000,000	3.125%	163,125	1,163,125
1/1/2026	0	--	147,500	147,500
7/1/2026	1,000,000	3.250%	147,500	1,147,500
1/1/2027	0	--	131,250	131,250
7/1/2027	1,000,000	3.375%	131,250	1,131,250
1/1/2028	0	--	114,375	114,375
7/1/2028	1,000,000	3.500%	114,375	1,114,375
1/1/2029	0	--	96,875	96,875
7/1/2029	1,000,000	3.625%	96,875	1,096,875
1/1/2030	0	--	78,750	78,750
7/1/2030	1,000,000	3.750%	78,750	1,078,750
1/1/2031	0	--	60,000	60,000
7/1/2031	1,000,000	3.875%	60,000	1,060,000
1/1/2032	0	--	40,625	40,625
7/1/2032	1,000,000	4.000%	40,625	1,040,625
1/1/2033	0	--	20,625	20,625
7/1/2033	1,000,000	4.125%	20,625	1,020,625

The yield is determined by finding the rate that sets the PV of Debt Service and the upfront proceeds equal to one another.

0.00 <==PV of Cashflows minus Upfront Proceeds

4.030% <==yield

Date	Principal	Coupon	Interest	Debt Service	PV of Debt Service
1/1/2024	0	--	15,000	15,000	14,704
7/1/2024	1,000,000	3.000%	15,000	1,015,000	975,296
	Issuance Date:	**7/1/2023**		**YIELD:**	4.0303%
PROCEEDS	**990,000**		**30,000**	**1,030,000**	**990,000**

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Poll question 5

Which is the following is INCORRECT:

A. A bond where coupon = yield is a par bond.

B. A bond where coupon > yield is a discount bond.

C. A bond where coupon > yield is a premium bond.

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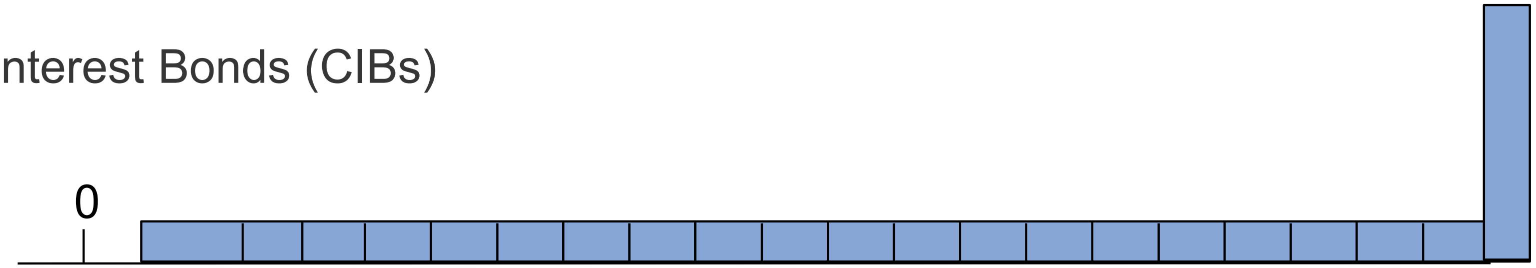
Terminology (cont'd)

- **Option:** The right, but not the obligation, to buy or sell a specific amount of a given asset at a specified price (the strike price) during a specified period of time.
  - **Call Option**: The right, but not the obligation, to buy an asset at a given price
    - Municipal bond issuers utilize call options to provide themselves flexibility to repay bonds prior to their stated maturity, i.e., through a refinancing/refunding.
  - **Put Option:** The right, but not the obligation, to sell a given asset at a specified price during a specific period. The writer of the put is obligated to buy the underlying asset and pay the strike price for it.
    - Put options are most commonly used in the municipal market for variable rate bonds. Buyers of variable bonds will commonly have a put option on these, where they can "put" the bonds back to the issuer and receive back their principal amount plus accrued interest.

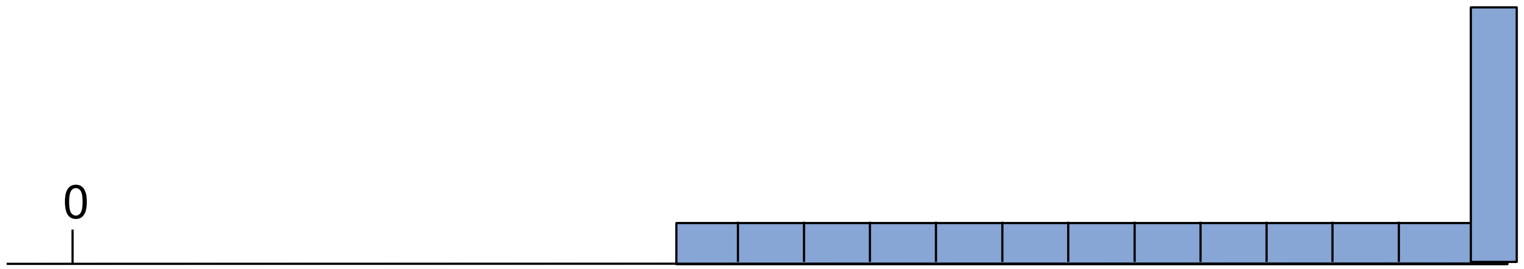
Types of Bonds

Current Interest Bonds (CIBs)



Capital Appreciation Bonds (CABs)/Zero Coupon Bonds

Convertible CABs/Deferred Interest Bonds (DIBs)



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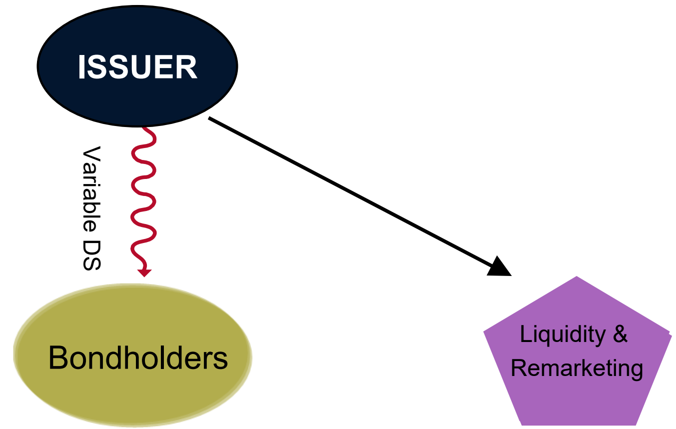
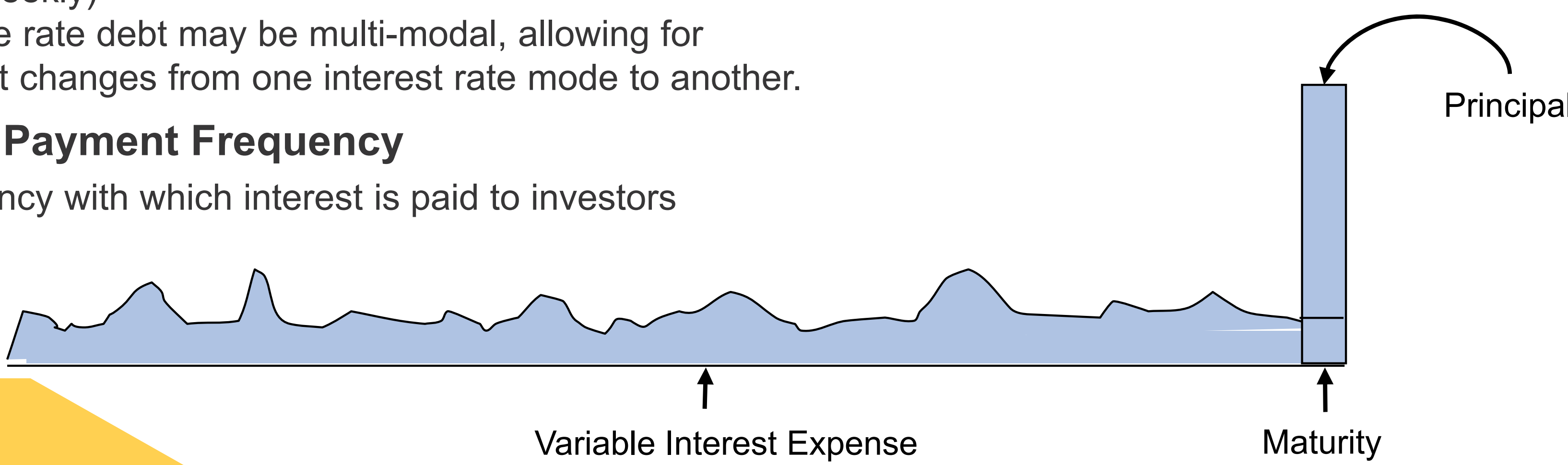
Mechanics of Variable Rate Debt

- **Variable Rate Bond**
  - Usually sold at par (no premiums!)
  - Nominal Maturity (pay down can be flexible)
  - Interest paid on a current basis at rates that are periodically reset
  - Bonds typically have a put option, allowing investors to put back bonds to issuers and receive their principal + accrued interest owed to them.
- **Interest Rate Reset Procedure**
  - Process by which interest rates are periodically reset on the variable rate debt
- **Interest Rate Mode**
  - Term for which interest rates are periodically reset (e.g., daily, weekly)
  - Variable rate debt may be multi-modal, allowing for discreet changes from one interest rate mode to another.
- **Interest Payment Frequency**
  - Frequency with which interest is paid to investors





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Thank you.

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